A STRUCTURAL APPROACH TO DESCRIBING CREEP IN A FIBER-BASED SOFT COMPOSITE

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Cemented fibrous materials (soft composites) are widely used, and it is important to forecast their physicomechanical properties.

Deformation and failure in nonwoven cemented materials are mainly described by a phenomenological approach [1-3], which does not fully incorporate the structure of the material, the properties of the fibers and bonding agent, the porosity, and so on, which would enable one to perform a purposive search for new materials with given properties. A structural model [4] is used here to examine creep in a soft composite on the basis of the linear viscoelastic behavior of the binding agent and fibers.

In that model, a biaxial state of stress arises on uniaxial stretching along the Oy axis by a stress σ, for which the static equilibrium equations [4] are

$$\sigma(1-\Omega_x)=\sigma_z\Omega_z\cos^2\alpha+\sigma_y\Omega_y,\quad \sigma_x\Omega_x+\sigma_z\Omega_z\sin^2\alpha=0 \tag{1}$$

and the compatibility conditions for the strains (taken as small) are

$$\begin{gathered}(1+\varepsilon_z)^2=(1+\varepsilon_x)^2\sin^2\alpha_0+(1+\varepsilon_y)^2\cos^2\alpha_0,\\ (1+\varepsilon_y)\operatorname{tg}\alpha=(1+\varepsilon_x)\operatorname{tg}\alpha_0\quad \forall\sigma\geqslant 0,\ \forall t\geqslant 0.\end{gathered} \tag{2}$$

In (1) and (2) we have used the [4] symbols, and we merely note that here and subsequently all quantities with subscripts x and y relate to the bonding agent and those with z to the fibers, while t is time.

To close (1) and (2), we assume that the fibers and the bonding agent obey a three-element rheological model:

$$\frac{d}{dt}\sigma_c+\nu_c\sigma_c=B_c\left(\frac{d}{dt}\varepsilon_c+\mu_c\varepsilon_c\right), \tag{3}$$

which is called a Kelvin body [5, 6]. Here B_c is the instantaneous modulus, $B_c\mu_c/\nu_c$ the long-time modulus for the corresponding element, and c = {x, y, z}. If $B_x \neq B_y$ (or $\nu_x \neq \nu_y$, $\mu_x \neq \mu_y$), we have a bonding agent whose rheological characteristics differ as between tension and compression.

System (1)-(3) completely describes the rheological behavior this unwoven material under uniaxial external stress. It is not possible to determine explicitly the time dependence of the strains ε_x and ε_y, e.g., in creep, since (1) and (2) are nonlinear. A numerical algorithm was set up involving integration by time step, which enables one to use a linear system of equations for the increments in the unknowns at each time step. We write the current values for the stresses and for the stresses and strains in the elements together with the reinforcement angle as

$$\begin{gathered}\sigma_{m+1}=\sigma_m+\Delta_m\sigma,\quad \sigma_{c,m+1}=\sigma_{c,m}+\Delta_m\sigma_c,\quad \varepsilon_{c,m+1}=\varepsilon_{c,m}+\Delta_m\varepsilon_c,\\ \alpha_{m+1}=\alpha_m+\Delta_m\alpha\quad \forall t\in[t_m,\ t_m+\Delta_m t],\ \Delta_m t=t_{m+1}-t_m,\ m=1,2,\ldots,\end{gathered} \tag{4}$$

in which $t_1 = 0$, σ_1, $\sigma_{c,1}$, $\varepsilon_{c,1}$, α_1 are fixed quantities derived from short-time loading, and c = {x, y, z}.

We substitute (4) into (1)-(3) and neglect quantities of the second order of smallness by comparison with the increments in all these quantities to get

$$\begin{gathered}(1-\Omega_x)\Delta_m\sigma=\Omega_y\Delta_m\sigma_y+\Omega_x\operatorname{ctg}^2\alpha_m\left(\frac{4\sigma_{x,m}\Delta_m\alpha}{\sin 2\alpha_m}-\Delta_m\sigma_x\right),\\ 0=\Delta_m\sigma_x\Omega_x+\Omega_z(\Delta_m\sigma_z\sin^2\alpha_m+\Delta_m\alpha\sigma_{z,m}\sin 2\alpha_m),\end{gathered}$$

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$$\lambda_{z,m}\Delta_m\varepsilon_z = \lambda_{x,m}\Delta_m\varepsilon_x \sin^2\alpha_0 + \lambda_{y,m}\Delta_m\varepsilon_y \cos^2\alpha_0, \quad \Delta_m\varepsilon_y \operatorname{tg}\alpha_m - \Delta_m\varepsilon_x \operatorname{tg}\alpha_0 + \Delta_m\alpha(\lambda_{y,m} + \lambda_{x,m}\operatorname{tg}\alpha_m \operatorname{tg}\alpha_0) = 0; \quad (5)$$

$$\frac{d}{dt}(\Delta_m\sigma_c) + \nu_c(\sigma_{c,m} + \Delta_m\sigma_c) = B_c\left[\frac{d}{dt}(\Delta_m\varepsilon_c) + \mu_c(\varepsilon_{c,m} + \Delta_m\varepsilon_c)\right] \quad (6)$$

($\lambda_{c,m} = 1 + \varepsilon_{c,m}$, $c = \{x, y, z\}$). As $\sigma_{c,m}$, $\varepsilon_{c,m}$, α_m are known at each step m + 1 in time, they are independent of time.

We use (5) and (6) for creep, i.e., for $\Delta_m\sigma = 0$, $\forall m$ and $\forall t \geqslant 0$, with the initial conditions

$$\Delta_m\sigma_c|_{t=t_m} = \Delta_m\varepsilon_c|_{t=t_m} = \Delta_m\alpha|_{t=t_m} = 0$$

to determine $\Delta_m\sigma_c$, $\Delta_m\varepsilon_c$, $\Delta_m\alpha$ $\forall t \in [t_m, t_m + \Delta_m t]$ as

$$X_m = A_{0,m} + \sum_{i=1}^{3} A_{i,m}\exp[K_{i,m}(t - t_m)],$$

in which $X_m = \{\Delta_m\sigma_c, \Delta_m\varepsilon_c, \Delta_m\alpha\}$; $A_{i,m} = \text{const}$.

Detailed calculations have been performed for $B_x = B_y = B$, $\mu_x = \mu_y = \mu$, $\nu_x = \nu_y = \nu$, $\mu_z = \nu_z = 0$, i.e., when the bonding agent is a linearly viscoelastic body (rheological parameters identical in tension and compression), while the fibers are linearly elastic. In this case, from (5) we determine $\Delta\sigma_x$, $\Delta\varepsilon_x$, $\Delta\varepsilon_y$, $\Delta\alpha$ in terms of $\Delta\sigma_y$, $\Delta\sigma_z$, $\Delta\varepsilon_z$. Then (6) can be transformed to a linear differential equation system for $\Delta\sigma_y$, $\Delta\sigma_z$, $\Delta\varepsilon_z$, whose characteristic polynomial is $I_{2,m}(K)^3 + I_{1,m}(K)^2 + I_{0,m}K$, where

$$I_{2,m} = \begin{vmatrix} a_{11,m} & a_{12,m} \\ a_{21,m} & a_{22,m} \end{vmatrix}, \quad I_{0,m} = \begin{vmatrix} a_{31,m} & a_{32,m} \\ a_{41,m} & a_{42,m} \end{vmatrix},$$

$$I_{1,m} = \begin{vmatrix} a_{11,m} & a_{12,m} \\ a_{41,m} & a_{42,m} \end{vmatrix} + \begin{vmatrix} a_{31,m} & a_{32,m} \\ a_{21,m} & a_{22,m} \end{vmatrix}.$$

The coefficients $a_{ij,m}$ ($i = \overline{1, 4}$, $j = 1, 2$) are not given because they are cumbersome and we merely note that they depend on ν, μ, $\lambda_{c,m}$, $\bar{\sigma}_{c,m}$, α_m, $\Gamma_1 = \Omega_y/\Omega_x$, $\Gamma_2 = \Omega_z/\Omega_x$, $\bar{B}_z = B_z/B$. Here and subsequently, the bars denote dimensionless quantities obtained by dividing by Young's modulus $B_x = B_y = B$. Then the increments at each step m are

$$\Delta_m\bar{\sigma}_y = A_{0,m} - A_{1,m}C_{2,m}\exp[K_{1,m}(\tau - \tau_m)] + A_{2,m}C_{1,m}\exp[K_{2,m}(\tau - \tau_m)],$$

$$\Delta_m\varepsilon_z = A_{3,m} + C_{2,m}\exp[K_{1,m}(\tau - \tau_m)] - C_{1,m}\exp[K_{2,m}(\tau - \tau_m)],$$

$$\Delta_m\bar{\sigma}_z = \bar{B}_z\Delta_m\varepsilon_z, \quad \Delta_m\bar{\sigma}_x = -\Gamma_1\Delta_m\bar{\sigma}_y - \Gamma_2\Delta_m\bar{\sigma}_z,$$

$$\Delta_m\varepsilon_x = \lambda_{x,m}[(\bar{B}_z \operatorname{ctg}^2\alpha_m/(2\bar{\sigma}_{z,m}) - 1/\lambda_{z,m})\Delta_m\varepsilon_z - \Gamma_1\Delta_m\bar{\sigma}_y/(2\bar{\sigma}_{x,m})],$$

$$\Delta_m\varepsilon_y = \lambda_{y,m}[(1/\lambda_{z,m} - \bar{B}_z/(2\bar{\sigma}_{z,m}))\Delta_m\varepsilon_z + \Gamma_1 \operatorname{tg}^2\alpha_m\Delta_m\bar{\sigma}_y/(2\bar{\sigma}_{x,m})],$$

$$\Delta_m\alpha = -\sin\alpha_m(\Gamma_1\Delta_m\bar{\sigma}_y/\cos\alpha_m + \Gamma_2\cos\alpha_m\Delta_m\bar{\sigma}_z)/(2\bar{\sigma}_{x,m}) \quad \forall\tau \in [\tau_m, \tau_{m+1}],$$

in which $\tau = \nu t$; $\tau_m = \nu t_m$, and the constants $C_{i,m}$ and $A_{j,m}$ ($i = 1, 2$, $j = \overline{0, 3}$) are defined via the $a_{ij,m}$, the characteristic roots, and the right-hand sides in (6). The solution has been used in a numerical algorithm. The time step $\Delta_m\tau = \nu\Delta_m t$ was chosen on the basis that all the increments are small, i.e.,

$$M = \max_{\tau\in[\tau_m,\tau_{m+1}]} \{|\Delta_m\bar{\sigma}_c|, |\Delta_m\varepsilon_c|, |\Delta_m\alpha|/\pi;\ c = x, y, z\} \leqslant \delta.$$

Here $0 < \delta \ll 1$ is a preset number, which is derived on the basis that quantities of the second order of smallness relative to the increments can be neglected. The computation terminates when one of the following constraints is met: 1) $\alpha_m = 0$ (with a given accuracy), and 2) $M < \varepsilon \ll \delta$. The first means that the fibers lie in the direction of the external load and the behavior of the composite is as for filaments, i.e., linearly elastic, while the second shows that the state of stress and strain hardly varies in time after a certain instant.

Figure 1 gives numerical results with $\delta = 10^{-2}$, $\varepsilon = 10^{-5}$, $\bar{B}_z = 20$, $\alpha_0 = \pi/3$, $\mu/\nu = 10^{-2}$ and

$$\bar{\sigma}_1 = 0{,}187,\ \bar{\sigma}_{x,1} = -0{,}386,\ \bar{\sigma}_{y,1} = 0{,}716,\ \bar{\sigma}_{z,1} = 0{,}168,\ \alpha_1 = 0{,}554; \quad (7a)$$

$$\bar{\sigma}_1 = 0{,}374,\ \bar{\sigma}_{x,1} = -0{,}515,\ \bar{\sigma}_{y,1} = 0{,}857,\ \bar{\sigma}_{z,1} = 0{,}378,\ \alpha_1 = 0{,}424, \quad (7b)$$

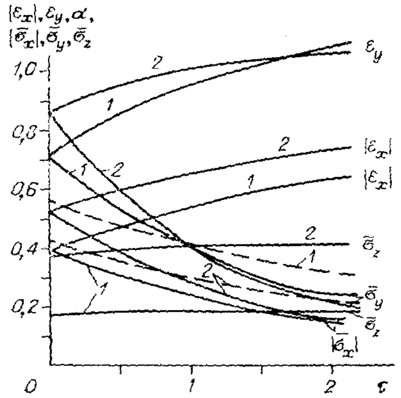

Fig. 1

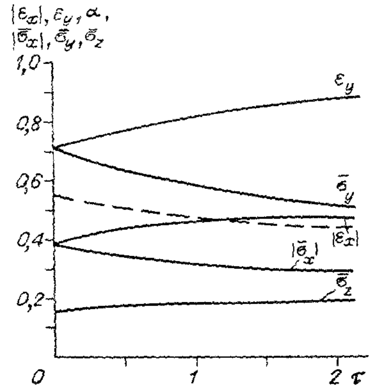

Fig. 2

which have been taken from [4] and correspond to short-time loading with linearly elastic behavior for the fibers and bonding agent. Curves 1 and 2 correspond to the parameters from (7a) and (7b). The dashed lines represent the change in the angle α between the intersecting filaments over time.

The results show that the instantaneous applied load in long-time strain has virtually no effect on ε_y, the longitudinal strain, and the longitudinal stress σ_y and transverse stress σ_x, but has a considerable effect on ε_x, α, and σ_z. The stresses σ_z in the fibers are dependent on time because of force redistribution between the elements, although they obey Hooke's law. However, it is readily shown that the dependence is unimportant (the same applies for the ε_z).

Numerical results for μ/ν of 0 to 10^{-2} show that the state of stress and strain is almost the same as that given in Fig. 1, whereas it differs considerably from it for $10^{-1} < \mu/\nu < 1$. Figure 2 shows results for the (7a) parameters with $\mu/\nu = 0.5$.

This model for the fiber base in a nonwoven material enables one to use the rheological characteristics of the binding agent and filaments to determine the creep under uniaxial stretching and also to determine the longitudinal and transverse strains and to correct for the finite change in the fiber orientation angle during the strain, which is particularly important for designing components made from synthetic materials.

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